Robust Welfare Guarantees for Decentralized Credit Organizations *

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Abstract

Rotating savings and credit associations (roscas) are informal financial organizations common in settings where communities have reduced access to formal financial institutions. In a rosca, a fixed group of participants regularly contribute small sums of money to a pool. This pool is then allocated periodically using lotteries or auction mechanisms. Roscas are empirically well-studied in the development economics literature. Due to their dynamic nature, however, roscas have proven challenging to examine theoretically. Theoretical analyses within economics have made strong assumptions about features such as the number or homogeneity of participants, the information they possess, their value for saving across time, or the number of rounds. This work presents an algorithmic study of roscas. We use techniques from the price of anarchy in auctions to characterize their welfare properties under less restrictive assumptions than previous work. Using the smoothness framework of [Syrgkanis and Tardos, 2013], we show that most common auction-based roscas have equilibrium welfare within a constant factor of the best possible. This evidence further rationalizes these organizations' prevalence. Roscas present many further questions where algorithmic game theory may be helpful; we discuss several promising directions.

1 Introduction

Rotating savings and credit associations (roscas) are informal financial organizations that are commonly used in low- and middle-income nations as well as many immigrant and refugee populations. These institutions serve as one mechanism for saving, credit, and insurance – among other forms of financial and social support – in settings where communities have reduced access to centralized financial institutions. Roscas often function as follows: a fixed number of participants, usually from similar socio-economic backgrounds, come together to contribute small sums of money to a pot in a periodic manner. At each meeting, the pot is then allocated to one participant who has not yet received it. The allocation may be determined by lottery or auction. Once a full round is completed and each participant has received a pot exactly once, the rosca may disband or start over for another full round. Recipients of the pot often use their influx of cash to invest, especially in more expensive durable goods such as farm equipment, appliances, or vehicles. As a result, this simple setup enables participants to effectively perform peer-to-peer lending: members who receive the pot earlier borrow from those who receive it later.

The prevalence of roscas cannot be overstated. They have been observed as endemic to communities on five continents; [Bouman, 1995a] alone documents roscas in 85 countries. [Bouman, 1995b] estimates that roscas account for about one-half of Cameroon's national savings, and [Aredo, 2004] estimates that over $1/6$ of households in Ethiopia's highlands participate in *ekub* – the region's variant of roscas. Roscas are also vital fixtures in immigrant populations, enabling members to find community and financial resources where such things might not otherwise be readily available.

Roscas' ubiquity has prompted nearly three decades of study by economists, starting with [Besley *et al.*, 1993]. Economic theory in particular has sought to explain the way roscas function as insurance, saving, and most importantly, lending devices for members. Theoretical studies of roscas have proved challenging for three key reasons. First, roscas are dynamic: participants make decisions in an online fashion, and condition their future decisions on past outcomes. Second, as participants are often uncertain about each others' financial needs, a theory of roscas must accommodate incomplete information. Finally, the sums of money in play in roscas are significant enough that agents' utilities are nonlinear. The standard economic approach of solving for auction equilibrium under any one of these phenomena in isolation is already challenging. More comprehensive approaches have only succeeded with aggressive simplifying assumptions.

This work initiates an algorithmic study of roscas. In particular, the theory of worst-case analysis of games, or *price of anarchy* provides tools for studying welfare properties of equilibria without directly solving for them. Using these tools, we investigate the allocative efficiency of roscas. That is, we ask how well these institutions coordinate saving and lending to agents whose opportunities to invest a rosca's pot may be heterogeneous across individuals and across time. We show that under a wide range of assumptions on both agents' values and the rules of the pot allocation protocol itself, roscas are able coordinate a groups' lending and borrowing in a way which approximately maximizes the group's to-

*Proofs of technical results can be found in the full paper at https://www.cs.cornell.edu/ red/abeberoscas.pdf tal utility. In this way, we provide a flexible analysis to match and explain the diversity of circumstances where roscas appear.

1.1 Our Contributions

To model saving and lending in roscas, we generalize the formulation of [Besley *et al.*, 1994]. We assume each agent seeks to purchase an investment such as a durable good, but only has means to do so upon winning a rosca's pot. We study the price of anarchy of auction-based roscas, where during each meeting, agents bid to choose a winner among those who have not yet received a pot. Agents must weigh the value of investing earlier against the utility-loss from spending their income to win. We give two main results (all proofs can be found in the full paper):

- We show that equilibria of the two most prevalent formats, ascending- and descending-price auctions (and their sealed-bid second- and first-price variants) maximize the total utility of participants up to a constant factor which degrades smoothly as agents' marginal values for money grows.
- We show that any rosca that uses a "reasonable" pot allocation protocol each period is guaranteed high equilibrium welfare. In other words, we give a sufficient condition on single-item auctions which guarantees that the rosca with that auction protocol has low price of anarchy.

We prove our main results using a variant of the smoothness framework developed in [Syrgkanis and Tardos, 2013]. The rosca setting requires us to adapt the framework to three challenges. First, payments in each stage of a rosca are typically redistributed among members, complicating the standard accounting. Second, participants' utilities depend on their payments (and rebates) in a nonlinear way. Finally, the sequential nature of roscas doesn't lend itself to standard smoothness composition arguments.

2 Model and Preliminaries

2.1 Formal Model for Roscas

We first present a typical model of roscas, following [Besley *et al.*, 1993], [Besley *et al.*, 1994]. A rosca for n agents takes place over n discrete time periods. Each period, two things occur. (1) Each agent pays an amount p_0 into a common pot. (2) A winner for the np_0 units of money in the pot is decided among those who have not yet won. The payments p_0 into the pot are decided ex ante, during the formation of the rosca. We will not model the process of selecting p_0, instead taking p_0 as given (cf. [Klonner, 2001]). The selection process (2) varies widely from rosca to rosca, and can involve a fully random lottery, allocation based on social status, or some form of auction. We focus on the latter due to their prevalence (see [Ardener, 1964]), and the analytical challenge they pose. In roscas that allocate by auction, we assume payments are distributed evenly amongst the non-winning members as a third step in the process, as is common in practice. We refer to this latter process as the *internalized seller*.

Roscas give individuals without access to banking a way to save for substantial investments such as large durable goods. We assume each agent desires to pay or a one-time investment, yielding for agent i a utility stream of ξ_i^t units at period t if and only if the agent has invested in the past. We follow [Besley *et al.*, 1993] and assume the cost of the investment is homogeneous across agents, and equal to the pot amount np_0. However, we allow ξ_i^t to vary across time and across agents. We further assume each agent has concave, nonnegative utility U for wealth consumption each period. Each period t, each agent has initial wealth level w. After paying p_0 into the pot and making additional payments p_i^t to the auction mechanism, the agent consumes their remaining wealth for period t utility $U(w - p_0 - p_i^t)$. Let $\mathbb{I}_i^t$ indicate whether the agent has invested by time t. An agent's utility at time t is thus $U_i(w_i - p_0 - p_i^t) + \xi_i^t \mathbb{I}_i^t$.

We study the utilitarian social welfare of the rosca, given by

$$\text{WELF} = \sum\nolimits_i \sum\nolimits_t \xi_i^t \mathbb{I}_i^t + \sum\nolimits_i \sum\nolimits_t U(w - p_0 - p_i^t).$$

As our benchmark, we consider the optimal ordering for pot allocation, i.e. one-to-one function j^* from agents to time periods maximizing the sum of agents' utility streams.

$$\text{OPT} = \max_{j*:[n]\to[n]} \sum_i \sum_{t=j*(i)}^{n} \xi_i^t + n^2 U(w - p_0),$$

and we seek to measure the worst-case approximation ratio between WELF and OPT.

We assume U is homogenous. This is valid in practice: participants in roscas tend to possess similar socioeconomic status, and thus it is reasonable to assume similar utility for wealth see [Mequanent, 1996], [Calomiris and Rajaraman, 1998]. (Note that similar socioeconomic status doesn't preclude varying investment opportunities or needs for large outlays of cash, and hence the ξ_i^t may yet be heterogeneous). We assume the slope of U is bounded. This matches the observation that investments from roscas tend to be significant (hence $U'(x) := 1$ is unreasonable), but not so large as to dwarf individuals' livelihoods [Aredo, 2004]. Also since multiplicative approximation guarantees only improve as $U(w - p_0)$ grows. It will therefore be entirely without loss to take $U(w - p_0) = 0$.

2.2 Roscas as Auctions

We now give definitions necessary to study the pot allocation procedure and define equilibrium in roscas. In the process, we will recast roscas in more standard auction-theoretic notation.

A *multi-round allocation setting* consists of n agents m items to be allocated, one per period. An allocation consists of a mapping from the items to the agents. Each agent has a real-valued valuation $v_i = (v_i^1, \ldots, v_i^m)$ over the items, and agents are unit-demand: their value for a set S of items is $\max_{j \in S} v_i^j$. Let $\mathcal{V}_i$ be the set of possible valuations of agent i. An outcome in a multi-round mechanism is an allocation x and payment vector p. Allocations are given by $x = (x_1, \ldots, x_n)$, where each $x_i = (x_i^1, \ldots, x_i^n)$ is an indicator vector, and $x_i^j = 1$ if and only if i receives item j.

Denote by $\mathcal{X}$ the space of feasible allocations. Payments are of the form $p_i = (p_i^1, \ldots, p_i^m)$, with each agent making payments each period. We assume agents' utilties to be additively separable with convex disutility function C for payments:

$$u_i^{v_i}(x_i, p_i) = v_i \cdot x_i - \sum\nolimits_j C(p_i^j).$$

The rosca setting maps into this framework in the following way: each pot in a rosca is a distinct item. An agent i with utilities $\xi_i^1, \ldots, \xi_i^n$ then has value $v_i^j = \sum_{k=j}^n \xi_i^k$ for winning the jth pot (and with later pots providing no additional value). Note that in this formulation, v_i^j is decreasing in j. The disutility function C for payments in a rosca can be taken to be $C(p_i^j) = -U(w - p_0 - p_i^j)$. Under these assumptions, we have $\sum_i u_i^{v_i}(x_i, p_i)$ is exactly equal to the social welfare given in Section 2.1.
We assume $C(0) = 0$, which is equivalent to $U(w - p_0) = 0$ and $\alpha \leq C'(p) \leq \beta$ where $\alpha, \beta > 0$.

In round $j \in [m]$ of a multi-round mechanism, the mechanism takes a profile of actions $a^j = (a_1^j, \ldots, a_n^j)$ and outputs an allocation $X^j(a^j)$ of item j and a profile of payments $P^j(a^j)$. The mechanism may condition X^j and P^j on previous rounds' actions. Typical pot allocation procedures in roscas resemble standard single-item auctions, with the additional restrictions that agents who won in previous rounds are ineligible for allocation, and that payments are redistributed among all agents.

We will consider mechanisms which are *individually rational* in each round; for any profile of actions in round j, the only agent with positive payments can be the winner of item j. We will further assume that the winner's payments are redistributed among the losers. For example, in round j of a rosca with a first-price rule, agents submit bids $b_1^j, \ldots b_n^j$. Among those who have not yet won the pot, the highest bidder i^* wins, and payments are $P_{i^*}^j = b_{i^*}^j$ and $P_i^j = b_{i^*}^j/(n-1)$ for $i \neq i^*$. We will show that not just first- and second-price auctions, but any "reasonble" (i.e. smooth, defined in Section 4) single-item auction can form the basis for an approximately-optimal rosca.

We study dynamic equilibria of roscas under incomplete information on agents' values for their investment opportunities. In auction notation, each agent draws their profile of values $v_i = (v_i^1, \ldots, v_i^n)$ from a distribution F_i, which is independent across agents, but correlated across rounds (in particular, decreasing across rounds). Agents can observe histories of play in past rounds, and hence a strategy s_i is a mapping s_i^j for each round j from values and histories of past play to actions/bids in round j. A profile of strategies $s = (s_1, \ldots, s_n)$ is a *Bayes-Nash equilibrium* if each agent's strategy maximizes their total utility across rounds, taken in expectation over other agents' values. In what follows, we fix an auction format for pot distribution, and bound the worst-case welfare approximation of roscas in the worst-case Bayes-Nash equilibrium, taken over all value distributions and equilibrium strategy profiles for those value distributions. That is, we study:

$$\max_{F,\, s \in \text{BNE}(F)} \frac{\mathbb{E}_{v\sim F}[\text{OPT}(v)]}{E_{v\sim F}[\sum_i u_i^{v_i}(X_i(s(v)), P_i(s(v)))]},$$

where $\text{OPT}(v) = \max_{x \in \mathcal{X}} \sum_i v_i \cdot x_i$. We call this ratio as the *price of anarchy* of the mechanism, per the standard terminology.

3 Warmup: Second-Price Roscas

The second-price rosca is the most prevalent form of rosca [Klonner, 2008]. Each round j, agents i who have not yet won an item[1] submit sealed bids b_i^j. The highest bidder wins the item and is charged the second-highest bid $b_{(2)}$. Importantly, losers are rebated $b_{(2)}/(n-1)$. We say a mechanism that redistributes payments in this way has an *internalized seller*.

Second-price-style mechanisms have unbounded price of anarchy due to overbidding equilibria. We therefore will restrict our consideration to equilibria without such pathologies.

Definition 3.1 (No Overbidding)**.** *A strategy profile s satisfies* no overbidding *if in every round j, agent i bids below $C^{-1}(v_i^j)$.*

In roscas, an agent i's value v_i^j for winning the pot in round j is their total income stream in subsequent rounds, $\sum_{k=j}^n \xi_i^k$. Consequently, v_i^j is nonincreasing in j. Our analysis will hold in any setting where this applies.

Definition 3.2 (Nonincreasing Values)**.** *A value vector v_i in a multi-round allocation setting is* nonincreasing with time *if $v_i^j \leq v_i^{j'}$ for all $j > j'$.*

Our analysis will proceed in the spirit of the weak smoothness framework of [Syrgkanis and Tardos, 2013]: for each bidder, we will construct a deviation strategy which exhibits a tradeoff between their utility and the bids of other agents in the auction. Our tradeoff will hold in every deterministic profile of actions and for every value profile. We can then extend the deviation to derive a welfare result in Bayes-Nash equilibrium.

Lemma 3.3. *For any value profile v and action profile a of a second-price rosca, there exists a deviation action $a_i^*(v, a_i)$ for each agent i such that the following inequality holds:*

$$\sum_i u_i^{v_i}(a_i^*(v, a_i), a_{-i}) \geq \text{OPT}(v) - 2\sum_j C\big(b_{(1)}^j\big), \quad (1)$$

where $b_{(1)}^j$ denotes the highest bid in round j under action profile a.

Theorem 3.4. *Assuming no overbidding and nonincreasing values, the price of anarchy for the second-price rosca is at most $1 + 2\beta/\alpha$ in Bayes-Nash equilibrium with independently distributed values across agents.*

The guarantee in Theorem 3.4 depends on the shape of the disutility function C, but degrades smoothly away from a price of anarchy of 3 as the ratio of C's slopes varies away from 1. Note that we require independence across agents, but not across the values for a particular agent. In fact, the nonincreasing values assumption rules the latter sort of independence out almost always.

[1] We refer to items in our abstraction to multi-item auctions and pots in the rosca application interchangeably.

4 Smoothness for Roscas

We now generalize the results from the previous section to reach a broader conclusion about roscas: as long as agents allocate the pot each period in a "reasonable" way, the rosca's welfare will be approximately optimal. So our definition of a "reasonable" mechanism is one that is smooth as in [Syrgkanis and Tardos, 2013]. However, the standard smoothness approach trades off revenue against agents' deviation utilities. In auctions with an internalized auctioneer, the latter quantity is always zero. We now give an adapted definition that considers multi-round allocation mechanisms, and only counts the payments made by winners each round. For mechanisms which are individually rational each round, the winners are the only agents generating payments to be redistributed. before rebates

Definition 4.1. *A multi-round allocation mechanism is* (λ, μ)-smooth before rebates *if for every for any valuation profile* $\times_i \mathcal{V}_i$ *and for any action profile* a *there exists a randomized action* $\mathbf{a}_i^*(v, a_i)$ *for each player* i *such that the following holds:*

$$\sum_i u_i^{v_i}(\mathbf{a}_i^*(v, a_i), a_{-i}) \geq \lambda \mathrm{OPT}(v) - \mu \sum_j \sum_i \max(P_i^j(a), 0).$$

In mechanisms without negative payments, Definition 4.1 matches the standard definition exactly. Moreover, mechanisms which are individually rational and (λ, μ)-smooth without an internalized seller are also smooth before rebates with the same parameters. We apply this result to the first-price auction in Section 6.

Lemma 4.2. *For any multi-round allocation mechanism that is individually rational each round and* (λ, μ)*-smooth, the same mechanism with an internalized seller is* (λ, μ)*-smooth before rebates.*

Lemma 4.3. *If an individually rational mechanism* $\mathcal{M}$ *with an internalized seller is* (λ, μ)*-smooth before rebates and agents have the option to withdraw, then if buyers have disutility for payments satisfying* $C(0) = 0$ *and* $C'(x) \in [\alpha, \beta]$ *for all* x*, value distributions are independent across agents, and values are nonincreasing, then the price of anarchy in Bayes-Nash equilibrium of* M *satisfying no overpayment is at most* $(\mu/\alpha + 1)/\lambda$

5 Round-Robin Composition

We now reduce the problem of designing a smooth rosca to the much simpler task of designing smooth single-item auctions. In the standard formulation of [Syrgkanis and Tardos, 2013], smooth mechanisms are closed under sequential composition with unit-demand agents. Roscas have a similar structure, with the key difference that agents who have won in earlier rounds are ineligible in later rounds. This difference imposes a surprising obstacle, and rules out a generalization of [Syrgkanis and Tardos, 2013]. As we saw in Section 3, the fact that values are nonincreasing over time will prove sufficient (and necessary) to obtain an extension theorem.

We first define composition for roscas. A rosca's pot allocation procedure for a given round is typically a single-item auction, given by M, with action space $\mathcal{A} = \times \mathcal{A}_i$, allocation rule X, and payment rule P. We assume M has a "withdraw" action $\perp$ which guarantees agents nonpositive payments. We allow $\perp$ to induce negative payments, as we allow mechanisms with an internalized seller, where winners' payments are redistributed to losers. A rosca can then be thought of as n copies of M composed in the following sense.

Definition 5.1 (Round-Robin Composition)**.** *Given single-item auction* $M = (\mathcal{A}, X, P)$ *with* n *agents, the* n-item round-robin composition *of* M *is a multi-round allocation mechanism for* n *items using the following procedure. Each round* j*, agents submit actions* $a^j = (a_1^j, \ldots, a_n^j)$*. The mechanism sets* $\underline{a}_i^j = a_i^j$ *for any* i *who have not yet won an item, and* $\underline{a}_i^j = \perp$ *for all* i *who have won an item previously. The mechanism then allocates item* j *to agents according to* M *applied to* $\underline{a}^j$ *and assigns payments for round* j *accordingly.*

We assume agents can condition their round j actions on play and outcomes in rounds $1, \ldots, j-1$. A multi-round allocation mechanism which is the round-robin composition of a smooth mechanism need not be smooth itself, due to two factors. First, agents can condition their actions on past play, and hence a small change in an agent's early bids might induce arbitrary behavior later. Second, there are action profiles where agents might have high value for later items, but win in early rounds. Smoothness deviations in these circumstances may not exist.

Theorem 5.2. *Let* M *be a* (λ, μ)*-smooth before rebates, individually rational single-item mechanism. Then if agents' values are nonincreasing with time, then the round-robin composition is* $(\min(1, \lambda), 1 + \mu)$*-smooth before rebates.*

6 Robust Welfare in Roscas

We now combine the pieces developed in the previous sections to show two sets of welfare guarantees in roscas. First, we give a general result: reasonable (i.e. smooth) single-item mechanisms make approximately-optimal roscas. Second, we give a specific guarantee for first-price/descending-price by analyzing the single-item mechanism without an internalized seller and applying the general result.

Theorem 6.1. *Let* M *be an individually rational single-item auction which is* (λ, μ)*-smooth when* C *satisfies* $C(0) = 0$ *and* $C'(x) \in [\alpha, \beta]$*. If values are nonincreasing, the* n*-round round-robin sequential composition of* M *with an internalized seller has price of anarchy at most* $((1 + \mu)\alpha^{-1} + 1)/\min(\lambda, 1)$ *in any Bayes-Nash equilibrium without overpayment.*

To instantiate this guarantee, we consider the single-item first-price auction. By Theorem 6.1, we need not consider more than one item or an internalized seller. However, smoothness guarantees from existing work only hold for quasilinear agents. We adapt the standard bound as follows:

Lemma 6.2. *If values are nonincreasing, the first-price rosca has price of anarchy at most* $(2/\alpha + 1)((1 - 1/e^{\beta})^{-1}\beta$ *in any Bayes-Nash equilibrium without overpayment.*

With quasilinear agents Lemma 6.2 yields a price of anarchy of $3e/(e-1)$.

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